

# The Constrained Conjugate Gradient Algorithm

J. A. Apolinário, Jr., *Member, IEEE*, M. L. R. de Campos, *Member, IEEE*, and C. P. Bernal O.

**Abstract**—Based on the condition for equivalence between linearly constrained minimum-variance (LCMV) filters and their generalized sidelobe canceler (GSC) implementations, we derive the new constrained conjugate gradient (CCG) algorithm. We discuss the use of orthogonal and nonorthogonal blocking matrices for the GSC structure and how the choice of this matrix may affect the relationship with the LCMV counterpart. The newly derived algorithm was tested in a computer experiment for adaptive multuser detection and showed excellent results.

**Index Terms**—Conjugate gradient algorithms, constrained adaptive filtering.

## I. INTRODUCTION

LINEARLY constrained adaptive filters have been used in many applications including adaptive beamforming with sensor arrays and blind adaptive interference cancellation in multuser mobile communication systems. The constrained version of the least mean square (LMS) algorithm (CLMS) was proposed in [1] for the minimization of the output-error energy of a finite impulse response (FIR) filter subject to a set of known linear constraints, i.e.,  $\min_w E[x^2]$  subject to  $\mathbf{C}^T \mathbf{w} = \mathbf{f}$ , where  $\mathbf{w}$  is the length  $M$  coefficient vector,  $x$  is the filter output error,  $\mathbf{C}$  is the  $M \times p$  constraint matrix, and  $\mathbf{f}$  is the length  $p$  gain vector. In [2], an alternative structure was presented whereby only a smaller set of coefficients are updated, which are confined to the subspace orthogonal to the space spanned by the constraint matrix  $\mathbf{C}$ . This structure, known as the generalized sidelobe canceler (GSC), is able to transform the linearly constrained minimization problem into an unconstrained minimization problem, and therefore can accommodate virtually any adaptation algorithm. Although the constrained algorithm and its GSC implementation are assumed to present identical steady-state performance [2] in a stationary environment, different choices of the blocking matrix  $\mathbf{B}$  such that  $\mathbf{B}^T \mathbf{C} = \mathbf{0}$  leads to different results. Moreover, this matrix determines the computational complexity of the adaptation algorithm implemented in the GSC structure. This paper revisits the condition of equivalence between a constrained adaptive filter and its GSC counterpart and uses this condition to introduce a new constrained algorithm, the constrained conjugate gradient (CCG) algorithm.

Manuscript received June 20, 2000. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. G. Rummeny. J. A. Apolinário, Jr. and C. P. Bernal O. are with the Facultad de Ingeniería Electrónica, Escuela Politécnica del Ejército, Sangolquí, Ecuador (e-mail: apolin@ieee.org).

M. L. R. de Campos is with the Programa de Engenharia Elétrica, COPPE/Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ 21-945-970 Brazil (campos@lps.sfi.br).

Publisher Item Identifier S 1070-9908/00/10143-X.

## II. PRELIMINARIES

The CLMS solution to the linearly constrained minimum-variance (LCMV) problem is given by [1]

$$\mathbf{w}(k+1) = \mathbf{P}\mathbf{w}(k) - \mu y(k) \mathbf{P} \mathbf{x}(k) + \mathbf{F} \quad (1)$$

where

$$\begin{aligned} \mathbf{F} &= \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{f}, \\ \mathbf{P} &= \mathbf{I} - \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \end{aligned}$$

projection matrix onto the subspace orthogonal to the subspace spanned by the constraint matrix, and the output signal;

$y(k)$  =  $\mathbf{w}^T(k) \mathbf{x}(k)$ , output signal.  
 $\mathbf{x}(k)$  is the input-signal vector containing present and past input-signal samples  $[x(k) \ x(k-1) \ \dots \ x(k-M+1)]^T$ . We recall the fact that although  $\mathbf{P}\mathbf{w}(k) + \mathbf{F}$  corresponds to  $\mathbf{w}(k)$  in infinite precision, the computation as in (1) is necessary in a limited-precision-arithmetic machine in order to avoid any drift from the constraint plane [1].

The GSC decomposes the coefficient vector by using a transformation matrix given by  $\mathbf{T} = [\mathbf{C}; \mathbf{B}]$  where  $\mathbf{B}$  is called blocking matrix, and it spans the null space of the constraint matrix  $\mathbf{C}$ . The GSC-transformed coefficient vector in  $\mathbf{w}(k) = \mathbf{T}\mathbf{w}(k)$  is partitioned as  $\mathbf{w}(k) = [\mathbf{w}_1^T; -\mathbf{w}_2^T]^T$ , where the upper part is constant and chosen such that  $\mathbf{C}\mathbf{w}_1$  corresponds to  $\mathbf{F} = \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{f}$ , and  $-\mathbf{w}_2 = \mathbf{w}_{\text{GSC}}(k)$  is updated according to an unconstrained adaptive filter such that the overall coefficient vector corresponds to  $\mathbf{w}(k) = \mathbf{F} - \mathbf{B}\mathbf{w}_{\text{GSC}}(k)$ .

The inverse of the GSC transformation matrix (guaranteed by linearly independent columns of  $\mathbf{B}$  and  $\mathbf{C}$ , and by  $\mathbf{B}^T \mathbf{C} = \mathbf{0}$  [3]) can be partitioned as  $\mathbf{T}^{-1} = [\mathbf{A}_1^T; \mathbf{A}_2^T]^T$  where  $\mathbf{A}_1 = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T$  and  $\mathbf{A}_2 = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T$ .

By replacing  $\mathbf{A}_1$  and  $\mathbf{A}_2$  in  $\mathbf{T}^{-1}$  and then in  $\mathbf{T}\mathbf{T}^{-1} = \mathbf{I}$ , we find another expression for the projection matrix  $\mathbf{P}$ , as obtained in [4]

$$\mathbf{P} = \mathbf{B}(\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T. \quad (2)$$

## III. EQUIVALENCE CONDITION REVISITED

In this section, we obtain the CLMS algorithm from its GSC implementation in order to find under which circumstances they are equivalent in infinite precision. The GSC coefficient-vector update equation using the LMS algorithm relates to the coefficient-vector update equation for the constrained LMS algorithm according to

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{F} - \mathbf{B}[\mathbf{w}_{\text{GSC}}(k+1)] \\ &= \mathbf{F} - \mathbf{B}[\mathbf{w}_{\text{GSC}}(k) + \mu y_{\text{GSC}}(k) \mathbf{x}_{\text{GSC}}(k)] \end{aligned} \quad (3)$$

# A Conjugate Gradient Algorithm For Analysis Of Variance

**Tobias Bleicker**



## **A Conjugate Gradient Algorithm For Analysis Of Variance:**

Embark on a transformative journey with Explore the World with is captivating work, Grab Your Copy of **A Conjugate Gradient Algorithm For Analysis Of Variance** . This enlightening ebook, available for download in a convenient PDF format , invites you to explore a world of boundless knowledge. Unleash your intellectual curiosity and discover the power of words as you dive into this riveting creation. Download now and elevate your reading experience to new heights .

<https://matrix.jamesarcher.co/files/Resources/HomePages/Linear%20Algebra%20With%20Applications%206th%20Edition%20Keith%20Nicholson.pdf>

## **Table of Contents A Conjugate Gradient Algorithm For Analysis Of Variance**

1. Understanding the eBook A Conjugate Gradient Algorithm For Analysis Of Variance
  - The Rise of Digital Reading A Conjugate Gradient Algorithm For Analysis Of Variance
  - Advantages of eBooks Over Traditional Books
2. Identifying A Conjugate Gradient Algorithm For Analysis Of Variance
  - Exploring Different Genres
  - Considering Fiction vs. Non-Fiction
  - Determining Your Reading Goals
3. Choosing the Right eBook Platform
  - Popular eBook Platforms
  - Features to Look for in an A Conjugate Gradient Algorithm For Analysis Of Variance
  - User-Friendly Interface
4. Exploring eBook Recommendations from A Conjugate Gradient Algorithm For Analysis Of Variance
  - Personalized Recommendations
  - A Conjugate Gradient Algorithm For Analysis Of Variance User Reviews and Ratings
  - A Conjugate Gradient Algorithm For Analysis Of Variance and Bestseller Lists
5. Accessing A Conjugate Gradient Algorithm For Analysis Of Variance Free and Paid eBooks
  - A Conjugate Gradient Algorithm For Analysis Of Variance Public Domain eBooks
  - A Conjugate Gradient Algorithm For Analysis Of Variance eBook Subscription Services

- A Conjugate Gradient Algorithm For Analysis Of Variance Budget-Friendly Options
- 6. Navigating A Conjugate Gradient Algorithm For Analysis Of Variance eBook Formats
  - ePub, PDF, MOBI, and More
  - A Conjugate Gradient Algorithm For Analysis Of Variance Compatibility with Devices
  - A Conjugate Gradient Algorithm For Analysis Of Variance Enhanced eBook Features
- 7. Enhancing Your Reading Experience
  - Adjustable Fonts and Text Sizes of A Conjugate Gradient Algorithm For Analysis Of Variance
  - Highlighting and Note-Taking A Conjugate Gradient Algorithm For Analysis Of Variance
  - Interactive Elements A Conjugate Gradient Algorithm For Analysis Of Variance
- 8. Staying Engaged with A Conjugate Gradient Algorithm For Analysis Of Variance
  - Joining Online Reading Communities
  - Participating in Virtual Book Clubs
  - Following Authors and Publishers A Conjugate Gradient Algorithm For Analysis Of Variance
- 9. Balancing eBooks and Physical Books A Conjugate Gradient Algorithm For Analysis Of Variance
  - Benefits of a Digital Library
  - Creating a Diverse Reading Collection A Conjugate Gradient Algorithm For Analysis Of Variance
- 10. Overcoming Reading Challenges
  - Dealing with Digital Eye Strain
  - Minimizing Distractions
  - Managing Screen Time
- 11. Cultivating a Reading Routine A Conjugate Gradient Algorithm For Analysis Of Variance
  - Setting Reading Goals A Conjugate Gradient Algorithm For Analysis Of Variance
  - Carving Out Dedicated Reading Time
- 12. Sourcing Reliable Information of A Conjugate Gradient Algorithm For Analysis Of Variance
  - Fact-Checking eBook Content of A Conjugate Gradient Algorithm For Analysis Of Variance
  - Distinguishing Credible Sources
- 13. Promoting Lifelong Learning
  - Utilizing eBooks for Skill Development
  - Exploring Educational eBooks
- 14. Embracing eBook Trends

- Integration of Multimedia Elements
- Interactive and Gamified eBooks

### **A Conjugate Gradient Algorithm For Analysis Of Variance Introduction**

Free PDF Books and Manuals for Download: Unlocking Knowledge at Your Fingertips In today's fast-paced digital age, obtaining valuable knowledge has become easier than ever. Thanks to the internet, a vast array of books and manuals are now available for free download in PDF format. Whether you are a student, professional, or simply an avid reader, this treasure trove of downloadable resources offers a wealth of information, conveniently accessible anytime, anywhere. The advent of online libraries and platforms dedicated to sharing knowledge has revolutionized the way we consume information. No longer confined to physical libraries or bookstores, readers can now access an extensive collection of digital books and manuals with just a few clicks. These resources, available in PDF, Microsoft Word, and PowerPoint formats, cater to a wide range of interests, including literature, technology, science, history, and much more. One notable platform where you can explore and download free A Conjugate Gradient Algorithm For Analysis Of Variance PDF books and manuals is the internet's largest free library. Hosted online, this catalog compiles a vast assortment of documents, making it a veritable goldmine of knowledge. With its easy-to-use website interface and customizable PDF generator, this platform offers a user-friendly experience, allowing individuals to effortlessly navigate and access the information they seek. The availability of free PDF books and manuals on this platform demonstrates its commitment to democratizing education and empowering individuals with the tools needed to succeed in their chosen fields. It allows anyone, regardless of their background or financial limitations, to expand their horizons and gain insights from experts in various disciplines. One of the most significant advantages of downloading PDF books and manuals lies in their portability. Unlike physical copies, digital books can be stored and carried on a single device, such as a tablet or smartphone, saving valuable space and weight. This convenience makes it possible for readers to have their entire library at their fingertips, whether they are commuting, traveling, or simply enjoying a lazy afternoon at home. Additionally, digital files are easily searchable, enabling readers to locate specific information within seconds. With a few keystrokes, users can search for keywords, topics, or phrases, making research and finding relevant information a breeze. This efficiency saves time and effort, streamlining the learning process and allowing individuals to focus on extracting the information they need. Furthermore, the availability of free PDF books and manuals fosters a culture of continuous learning. By removing financial barriers, more people can access educational resources and pursue lifelong learning, contributing to personal growth and professional development. This democratization of knowledge promotes intellectual curiosity and empowers individuals to become lifelong learners, promoting progress and innovation in various fields. It is worth noting that while accessing free A Conjugate Gradient Algorithm For Analysis Of Variance PDF

books and manuals is convenient and cost-effective, it is vital to respect copyright laws and intellectual property rights. Platforms offering free downloads often operate within legal boundaries, ensuring that the materials they provide are either in the public domain or authorized for distribution. By adhering to copyright laws, users can enjoy the benefits of free access to knowledge while supporting the authors and publishers who make these resources available. In conclusion, the availability of A Conjugate Gradient Algorithm For Analysis Of Variance free PDF books and manuals for download has revolutionized the way we access and consume knowledge. With just a few clicks, individuals can explore a vast collection of resources across different disciplines, all free of charge. This accessibility empowers individuals to become lifelong learners, contributing to personal growth, professional development, and the advancement of society as a whole. So why not unlock a world of knowledge today? Start exploring the vast sea of free PDF books and manuals waiting to be discovered right at your fingertips.

### FAQs About A Conjugate Gradient Algorithm For Analysis Of Variance Books

**What is a A Conjugate Gradient Algorithm For Analysis Of Variance PDF?** A PDF (Portable Document Format) is a file format developed by Adobe that preserves the layout and formatting of a document, regardless of the software, hardware, or operating system used to view or print it. **How do I create a A Conjugate Gradient Algorithm For Analysis Of Variance PDF?** There are several ways to create a PDF: Use software like Adobe Acrobat, Microsoft Word, or Google Docs, which often have built-in PDF creation tools. Print to PDF: Many applications and operating systems have a "Print to PDF" option that allows you to save a document as a PDF file instead of printing it on paper. Online converters: There are various online tools that can convert different file types to PDF. **How do I edit a A Conjugate Gradient Algorithm For Analysis Of Variance PDF?** Editing a PDF can be done with software like Adobe Acrobat, which allows direct editing of text, images, and other elements within the PDF. Some free tools, like PDFescape or Smallpdf, also offer basic editing capabilities. **How do I convert a A Conjugate Gradient Algorithm For Analysis Of Variance PDF to another file format?** There are multiple ways to convert a PDF to another format: Use online converters like Smallpdf, Zamzar, or Adobe Acrobats export feature to convert PDFs to formats like Word, Excel, JPEG, etc. Software like Adobe Acrobat, Microsoft Word, or other PDF editors may have options to export or save PDFs in different formats. **How do I password-protect a A Conjugate Gradient Algorithm For Analysis Of Variance PDF?** Most PDF editing software allows you to add password protection. In Adobe Acrobat, for instance, you can go to "File" -> "Properties" -> "Security" to set a password to restrict access or editing capabilities. Are there any free alternatives to Adobe Acrobat for working with PDFs? Yes, there are many free alternatives for working with PDFs, such as: LibreOffice: Offers PDF editing features. PDFsam: Allows splitting, merging, and editing PDFs. Foxit Reader:

Provides basic PDF viewing and editing capabilities. How do I compress a PDF file? You can use online tools like Smallpdf, ILovePDF, or desktop software like Adobe Acrobat to compress PDF files without significant quality loss. Compression reduces the file size, making it easier to share and download. Can I fill out forms in a PDF file? Yes, most PDF viewers/editors like Adobe Acrobat, Preview (on Mac), or various online tools allow you to fill out forms in PDF files by selecting text fields and entering information. Are there any restrictions when working with PDFs? Some PDFs might have restrictions set by their creator, such as password protection, editing restrictions, or print restrictions. Breaking these restrictions might require specific software or tools, which may or may not be legal depending on the circumstances and local laws.

### **Find A Conjugate Gradient Algorithm For Analysis Of Variance :**

~~linear algebra with applications 6th edition keith nicholson~~

**les grands auteurs en eacuteconomie**

~~level e unit 4 vocab answers~~

~~living religions mary pat fisher 7th~~

~~leadership research findings practice and skills by dubrin andrew j published by cengage learning 7th seventh edition 2012 paperback~~

**lean process measurement and lean tools techniques**

**literature 6e robert diyanni**

~~learn english 300 faster 69 tips to speak english like a native english speaker~~

~~level chart macmillan~~

~~libro origen dan brown rese as resumen y comentarios~~

~~lean lean bible six sigma 5s 3 manuscripts 1 bonus book lean thinking lean production lean manufacturing lean startup kaizen~~

~~libro actua como dama pero piensa como hombre gratis completo~~

~~learning teaching macmillan books for teachers jim scrivener~~

**les synonymes 1 barre l intrus de chaque liste**

~~libro de superacion de cesar guzman~~

### **A Conjugate Gradient Algorithm For Analysis Of Variance :**

GIS Tutorial 2: Spatial Analysis Workbook ... GIS Tutorial 2: Spatial Analysis Workbook provides hands-on exercises for

intermediate-level GIS users to build problem-solving and analysis skills. GIS Tutorial 2: Spatial Analysis Workbook, 10.1 Edition ... Jan 17, 2013 — This intermediate workbook helps ArcGIS users build problem-solving and spatial analysis skills. Solved: GIS Tutorial 2: Spatial Analysis Workbook 10.3x Tu... Aug 21, 2021 — I purchased the ebook titled GIS Tutorial 2: Spatial Analysis Workbook 10.3x , which directed me to the esri.com book resources section. GIS Tutorial 2: Spatial Analysis Workbook The GIS Tutorial 2: Spatial Analysis Workbook is a well written step-by-step guide with easy to understand directions and tutorials. Book 2 from the Esri ... GIS Tutorial 2 | Guide books - ACM Digital Library by DW Allen · 2010 · Cited by 122 — Updated for ArcGIS Desktop 10, GIS Tutorial 2: Spatial Analysis Workbook offers hands-on exercises to help GIS users at the intermediate level continue to ... GIS Tutorial 2: Spatial Analysis Workbook - David W. Allen GIS Tutorial 2: Spatial Analysis Workbook provides hands-on exercises for intermediate-level GIS users to build problem-solving and analysis skills. GIS Tutorial 2: Spatial Analysis Workbook / Edition 2 GIS Tutorial 2: Spatial Analysis Workbook provides hands-on exercises for intermediate-level GIS users to build problem-solving and analysis skills. GIS tutorial 2 : spatial analysis workbook Summary. GIS Tutorial 2: Spatial Analysis Workbook provides hands-on exercises for intermediate-level GIS users to build problem-solving and analysis skills. GIS tutorial 2 : spatial analysis workbook Details · "For ArcGIS 10.1." · Originally published as: GIS tutorial II : spatial analysis workbook. 2009. · Includes index. · Accompanying DVD-ROM contains ... GIS Tutorial 2 - Spatial Analysis Workbook | PDF GIS Tutorial 2 - Spatial Analysis Workbook - Free ebook download as PDF File (.pdf) or read book online for free. GUIA PARA EL MANEJO DE ARGIS. 168 Sample Report Card Comments (Plus a Printable ... Nov 17, 2023 — Use these sample report card comments for report cards. We have comments for everything you'd want to tell parents about their student. 107 Report Card Comments to Use and Adapt Mar 17, 2023 — For example, you'll be able to take a 1st grade number sense comment ... Write 50 report card comments for students studying social studies in 5th ... 125 Report Card Comments - Education World It's report card time and you face the prospect of writing constructive, insightful, and original comments on a couple dozen report cards or more. Report Card comments Feb 10, 2008 — I googled "report card comments" and found a few neat sites with good examples. Here is a list of just about every comment imaginable. 15 Best Report Card Comments Samples 6 days ago — Example Report Card Comments · 1. \_\_\_\_\_ is an excellent student who always comes prepared for class. · 2. \_\_\_\_\_ is a smart and inquisitive ... 100 Report Card Comments You Can Use Now 100 report card comments you can use right now! Provide valuable and insightful feedback to parents and students with this comprehensive list. 325 amazing report card comments and remarks to save ... Apr 4, 2022 — What's more difficult than creating your student report cards? Writing unique and meaningful report card comments. Creating report card ... Second quarter Report Card Comments. First Quarter. It has been a joy getting to know \_\_\_\_\_ this quarter. She is a kind and caring student who is always willing to ... 180 Report Card Comments for All Grades in 2023 May 4, 2023 — 180 Report Card Comments. Positive and negative report card comment demo. Copy to Text Blaze. enthusiastic positive always ready

enthusiasm 2 Exploring English, Level 1 by Harris, Tim This fully illustrated six-level series will set your students on the road to English language fluency. Exploring English, written by Tim Harris and illustrated ... Exploring English, Level 1: Workbook by Harris, Tim This fully illustrates six-level series will set your students on the road to English language fluency. Exploring English teaches all four language skills right ... Exploring English 1 book by Tim Harris This fully illustrated six-level series will set your students on the road to English language fluency. Exploring English , written by Tim Harris and ... Exploring English - Tim Harris, Timothy A. Harris, Allan Rowe This fully illustrated six-level series will set your students on the road to English language fluency. Exploring English, written by Tim Harris and ... Exploring English, Level 1 by Allan Rowe and Tim Harris ... This fully illustrated six-level series will set your students on the road to English language fluency. Exploring English , written by Tim Harris and ... Exploring English, Level 1 - Harris, Tim; Rowe, Allan Exploring English, written by Tim Harris and illustrated by Allan Rowe, teaches all four language skills right from the start, and gives students a wealth of ... Exploring English, Level 6 / Edition 1 This fully illustrated six-level series will set your students on the road to English language fluency. Exploring English, written by Tim Harris. Exploring English, Level 1: Workbook by Tim Harris This fully illustrates six-level series will set your students on the road to English language fluency. Exploring English teaches all four language skills right ... Exploring English 1 Teacher's Resource... book by Tim Harris This comprehensive six-part series teaches all four language skills from the start. The tapes use a broad range of characters and real-life situations, ... Exploring English, Level 1 Workbook Buy Exploring English, Level 1 Workbook by Tim Harris, Allan Rowe (ISBN: 9780201825930) online at Alibris. Our marketplace offers millions of titles from ...