

# Optimum Multiuser Detection Is Tractable for Synchronous CDMA Systems Using $M$ -Sequences

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**Abstract**—The optimum multiuser detection problem was shown to be NP-hard, i.e., its computational complexity increases exponentially with the number of users [1], [2]. In this letter, we show that the optimum multiuser detection problem for a synchronous code-division multiple access (CDMA) system is equivalent to the minimum capacity cut problem in a related network and propose an optimum multiuser detection algorithm with polynomial computational complexity for a certain class of signature sequences. The minimum cut problem is solvable in polynomial time if the capacities of the links not incident to source and sink are nonnegative. This condition in the optimum detection problem is equivalent to all cross correlations between the signature sequences of the users being negative. One example of such set of signature sequences is obtained when shifted versions of the maximal length sequences (or  $m$ -sequences) are used. In this case the cross correlation between users  $i$  and  $j$  is given as  $\Gamma_{ij} = -1/G$  for all  $i, j$ , where  $G$  is the processing gain.

**Index Terms**—CDMA, optimum multiuser detection.

## I. INTRODUCTION

**I**N CODE-DIVISION multiple access (CDMA) systems users are assigned unique signature waveforms which they use to modulate their information bits. Let the signature sequence of the  $i$ th user be  $s_i(t)$  for  $t \in [0, T]$  where  $T$  is the bit duration. The received signal for a synchronous CDMA system with binary phase-shift keying (BPSK) modulation is given by

$$r(t) = \sum_{i=1}^N A_i \alpha_i s_i(t) + n(t) \quad (1)$$

where  $A_i$  and  $\alpha_i$  are received amplitude and the transmitted bit ( $\pm 1$  equiprobably) of the  $i$ th user and  $n(t)$  is the additive white Gaussian noise (AWGN) process with power spectral density  $\sigma^2$ . The received signal vector at the output of the conventional receivers is given by

$$\mathbf{y} = \mathbf{\Gamma} \mathbf{A} \mathbf{a} + \mathbf{n}. \quad (2)$$

The vector  $\mathbf{y}$  is a sufficient statistics for the multiuser detection problem. In (2),  $\mathbf{\Gamma}$  is a nonnegative definite matrix where  $\Gamma_{ij} = \int_0^T s_i(t) s_j(t) dt$ ,  $\mathbf{A}$  is a diagonal matrix containing the received amplitudes of the users with  $A_{ii} = A_i$ ,  $\mathbf{a}$  is the vector

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containing the information bits of the users and  $\mathbf{n}$  is a Gaussian random vector with auto covariance matrix  $E[\mathbf{n}\mathbf{n}^T] = \sigma^2 \mathbf{I}$ .

The aim of the multiuser detection is to recover the information bits transmitted by the users in this multiaccess environment. Optimum multiuser detection [1] is based on the maximum likelihood criteria. The optimum multiuser detector chooses  $\mathbf{a}^*$  as the transmitted bit vector if for  $\mathbf{a} = \mathbf{a}^*$  the conditional probability density of  $\mathbf{y}$  given  $\mathbf{a}$  is maximized. Denoting the probability density function of  $\mathbf{n}$  by  $f_{\mathbf{n}}(\cdot)$ , the optimum detection problem is given as

$$\begin{aligned} \mathbf{a}^* &= \underset{\mathbf{a} \in \{-1, 1\}^N}{\text{arg max}} f_{\mathbf{n}}(\mathbf{y} - \mathbf{\Gamma} \mathbf{A} \mathbf{a}) \\ &= \underset{\mathbf{a} \in \{-1, 1\}^N}{\text{arg max}} \mathbf{a}^T \mathbf{R} \mathbf{a} - 2 \mathbf{a}^T \mathbf{A} \mathbf{y} \end{aligned} \quad (3)$$

where  $\mathbf{R} = \mathbf{A} \mathbf{\Gamma} \mathbf{A}$  with  $R_{ij} = A_i A_j \Gamma_{ij}$ . We can convert (3) to a 0-1 programming problem by introducing a vector  $\mathbf{b}$  where  $\mathbf{b} = (\mathbf{a} + \mathbf{u})/2$  and  $\mathbf{u}$  is an  $N$ -dimensional vector of all ones,  $\mathbf{u} = [1 \ 1 \ 1 \ \dots \ 1]^T$  as

$$\mathbf{b}^* = \underset{\mathbf{b} \in \{0, 1\}^N}{\text{arg max}} \mathbf{b}^T \mathbf{R} \mathbf{b} - \mathbf{b}^T \mathbf{y} \quad (4)$$

where  $\mathbf{y} = \mathbf{R} \mathbf{u} + \mathbf{A} \mathbf{y}$ . Note that the solutions of (3) and (4) are related by the one-to-one relationship  $a_i^* = 2b_i^* - 1$ .

## II. NETWORK PRELIMINARIES

Consider a network  $G = [V, A]$  with vertices  $V = \{0, 1, \dots, N+1\}$  and arcs  $A$ . For any two vertices  $i$  and  $j$  in  $G$ ,  $c_{ij}$  denotes the capacity of the arc connecting  $(i, j)$ . Let the nodes 0 and  $N+1$  represent the source and the sink, respectively. A cut separating 0 and  $N+1$  is a partition of the nodes  $(S, \bar{S})$  where  $0 \in S$ ,  $N+1 \in \bar{S}$ ,  $S \cup \bar{S} = V$ , and  $S \cap \bar{S} = \emptyset$ . The capacity of the cut  $(S, \bar{S})$  is given by [3]

$$C(S, \bar{S}) = \sum_{i \in S} \sum_{j \in \bar{S}} c_{ij} \quad (5)$$

The minimum cut separating nodes 0 and  $N+1$  is defined to be the cut separating nodes 0 and  $N+1$  and having the minimum capacity.

In [4] it was shown that any cut separating nodes 0 and  $N+1$  can be represented by a vector  $(1, b_1, b_2, \dots, b_N, 0)$  where  $b_i \in \{0, 1\}$  for  $i = 1, \dots, N$  is an indication for membership in  $S$ . That is  $S = \{i | b_i = 1\}$  and  $\bar{S} = \{i | b_i = 0\}$ . It was also shown in [4] that the capacity of the cut  $(S, \bar{S})$  is given by

$$C(b) = \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} c_{ij} b_i (1 - b_j) \quad (6)$$

# Computational Complexity Of Optimum Multiuser Detection

**Tao Wei**



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